Calculate the matrix product

$$B(2)B(3)\cdots B(n).$$

Solutions

- **5307:** *Proposed by Haishen Yao and Howard Sporn, Queensborough Community College, Bayside, NY*

Solve for x:

$$\sqrt{x^{15}} = \sqrt{x^{10}-1} + \sqrt{x^5-1}.$$

Solution 1 by Arkady Alt, San Jose, CA

Let $a = \sqrt{x^{10}-1}$ and $b = \sqrt{x^5-1}$ then

$$\begin{aligned}
x^5 &= b^2+1,\ \ x^{10} = a^2+1, \\
x^{15} &= x^{10}\cdot x^5 = \left(a^2+1\right)\left(b^2+1\right) \text{ and therefore,} \\
\sqrt{\left(a^2+1\right)\left(b^2+1\right)} &= a+b \iff \\
\left(a^2+1\right)\left(b^2+1\right) &= (a+b)^2 \iff \\
(ab-1)^2 &= 0 \iff \\
ab = 1.
\end{aligned}$$

Also we have

$$x^{10} = \left(x^5\right)^2 \implies a^2+1 = \left(b^2+1\right)^2 \iff b^4+2b^2 = a^2 \iff b^6+2b^4 = a^2b^2.$$

Since $ab = 1$ then

$$\begin{aligned}
b^6+2b^4-1 &= 0 \iff \\
\left(b^2+1\right)\left(b^4+b^2-1\right) &= 0 \iff \\
b^4+b^2-1 &= 0 \iff \\
b^2 &= \frac{-1+\sqrt{5}}{2}. \text{ Hence,} \\
x^5 &= b^2+1 \\
&= \frac{-1+\sqrt{5}}{2}+1 \\
&= \frac{1+\sqrt{5}}{2} \iff x = \sqrt[5]{\frac{1+\sqrt{5}}{2}}.
\end{aligned}$$